III (7)

## Non-linear PDE of order 1 —

$f^n$ $\phi(x, y, z, a, b) = 0$

$x, y$: independent variables; $z$: dependent variable; $a, b$: consts

### Standard Form 1 — $f(p, q) = 0$, No $x, y, z$

**Ex. 1** Solve $p^2 + q^2 = n^2$

**Sol.** C.I. (Complete Integral) is $z = ax + by + c$ —— (1)

Put $p = a$, $q = b$ (find $b$ in terms of $a$)

$a^2 + b^2 = n^2$ or $b = \sqrt{n^2 - a^2}$

Put $b$ in (1)

$\therefore$ C.I. $z = ax + \sqrt{n^2 - a^2}\, y + c$ —— (2)

**General Integral (G.I.)** —

Let $c = \phi(a)$ ; $\phi$ is arbitrary $f^n$

(2) becomes $z = ax + (\sqrt{n^2 - y^2})\, y + \phi(a)$ —— (3)

diff (3) wrt 'a' $\Rightarrow$

$$0 = x + \frac{1}{2}(n^2 - a^2)^{\frac{1}{2} - 1} \cdot (-2a) \cdot y + \phi'(a)$$

$$0 = x - \frac{a \cdot y}{\sqrt{n^2 - a^2}} + \phi'(a) \quad \text{—— (4)}$$

G.I. obtained by eliminating 'a' from (3) & (4)

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Ex. 9. **Standard Form 2**

Solve $z = px + qy + pq$

Sol. C.I. is $z = ax + by + ab$ —— (1)
(Put $p = a$, $q = b$)

G.I. let $b = \phi(a)$ ; $\phi$ is arbitrary $f^n$

$\therefore$ from (1), $z = ax + y\phi(a) + a\phi(a)$ —— (2)

diff (2) partially w.r.t 'a'

$0 = x + y\phi'(a) + a\phi'(a) + \phi(a)$ —— (3)

G.I. obtained by eliminating 'a' from (2) & (3)

**Singular Integral (S.I.)** –

Diff. partially (1) w.r.t a & b

$0 = x + b$ —— (4)

$0 = y + a$ —— (5)

Eliminate a, b between (1), (4), (5)

(Put $a = -y$, $b = -x$ in (1)) $\Rightarrow$

$z = (-y)x + (-x)y + (-y)(-x)$

$= -xy$

$\therefore$ S.I. $= -xy$

P407 Ex. 14.

Standard Forms III (9)

Find C.I. of $z = pq$

Sol. It is of form $f(z, p, q) = 0$

Let $z = f(X)$ where $X = x + ay$ & so

put $\frac{dz}{dX}$ for $p$ & $\frac{a\,dz}{dX}$ for $q$

Given eqn becomes

$$z = \left(\frac{dz}{dX}\right) \cdot \left(\frac{a\,dz}{dX}\right) = a\left(\frac{dz}{dX}\right)^2$$

or $$\frac{dz}{dX} = \pm \frac{\sqrt{z}}{\sqrt{a}}$$

$$\pm\sqrt{a}\, dz \cdot z^{-1/2} = dX$$

Integrate: $$\pm 2\sqrt{az} = X + c$$

$$\pm 2\sqrt{az} = (x + ay) + c$$

C.I. : $$(x + ay + c)^2 = 4az$$

# Standard Form 4



P. 412 EX23 Find C.I. & G.I. of

$$\sqrt{p}+\sqrt{q}=2x$$

Sol. $\sqrt{p}-2x=-\sqrt{q}=a$ (say)

C.I. $\therefore p=(2x+a)^2,\ q=a^2$

$$dz=p\,dx+q\,dy$$
$$=(2x+a)^2dx+a^2dy$$

Integrate, $\therefore z+b=\dfrac{(2x+a)^3}{3\cdot 2}+a^2y$ is C.I. — (2)

G.I. Put $b=\phi(a)$ in (2),

$$z+\phi(a)=\frac{(2x+a)^3}{6}+a^2y \quad (3)$$

Diff partially (3) wrt. 'a'

$$\phi'(a)=\frac{(2x+a)^2}{2}+2ay \quad (4)$$

G.I. obtained by eliminating 'a' from (3) & (4).